THE MODIFIED HOPKINS-STRYKER EQUATION

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## Abstract

From the original work of Hopkins-Stryker in 1948, through its development by Beranek in 1949, and its use by Davis in 1968 and Boner in 1969 in acoustic gain calculations, the Hopkins-Stryker equation has proven highly useful to a myriad of users. In recent years this versatile tool has been modified in the light of measurements by Peutz and Davis to account for multiple sources, semi-reverberant spaces, modifiers of critical distance, and various electroacoustic modifiers of the ratio of direct-to-reverberant sound. This paper is a thorough discussion of these modifications and their proper application in acoustic calculations.

## Introduction

The Hopkins-Stryker equation and its derivatives are based on the same assumptions used by Sabine in his classical study of reverberation. Sabine predicted a stochastic process in an ergodic enclosure (i.e., randomly mixing, homogeneous space). It is this description that qualifies a "reverberant sound field" as an entity distinct from a discrete reflection or a "limited train" of discrete reflections.

It was in this context that the equation called "Hopkins-Stryker" came into being with separate terms to account for the direct sound field and the reverberant sound field.

Measurements and empirical calculations have provided a further term, when required, for semi-reverberant situations. With these "cavets" let's proceed to the equation and its variations.

## Modifying the Hopkins-Stryker Equation

I. BASIC EQUATION

$$L_T = L_W + 10\ \mathrm{LOG}\left(\frac{Q(Me)}{4\pi(D_X)^2} + \frac{4N}{S\bar{a}\ (Ma)}\right) + 10.5$$

Use when $\Delta dB \leqq 1.0$ or less

II. DIRECT SOUND LEVEL

$$L_D = L_W + 10\ \mathrm{LOG}\left(\frac{Q(Me)}{4\pi(D_X)^2}\right) + 10.5$$

Use when $\Delta dB$ is $\geqq$ than 5.0

III. REVERBERANT SOUND LEVEL

$$L_R = L_W + 10\ \mathrm{LOG}\left(\frac{4N}{S\bar{a}\ (Ma)}\right) + 10.5$$

Use when $\Delta dB << 0.5$

IV. ACTUAL SOUND LEVEL

$$L_{act} = L_W + 10\ \mathrm{LOG}\left(\frac{Q}{4\pi(D_C)^2}\right) + \left(0.734\left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right)\left(\mathrm{LOG}\ \frac{D_C}{D_X > D_C}\right)\right) + 10.5$$

Use when $\Delta dB$ falls between 1.0 and 5.0 dB

Where: $L_T$ is the total sound pressure level in dB at $D_X$ (ref. 20 upa)

$L_D$ is the direct sound pressure level in dB at $D_X$ (ref. 20 upa)

$L_R$ is the reverberant sound pressure level in dB at $D_X$ (ref. 20 upa)

$L_{act}$ is the actual total sound pressure level in dB that occurs in semi-reverberant sound fields at $D_X > D_C$ (ref. 20 upa)

$L_W$ is the sound power level in dB for the device providing $L_D$ at $D_X$ (ref. $10^{-12}$ watt)

$\Delta dB$ is the number of dB $L_{act}$ is below the $L_T$ predicted by the basic Hopkins-Stryker equation at $2D_C$

$$\Delta dB = 0.221^{*}\left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right)$$

*Metric (S.I.) 0.4
($\Delta dB > 6$ dB = 6 dB)

Q is the directivity factor (dimensionless) for the device providing $L_D$ at $D_X$

$D_X$ is the distance in feet from the source to where $L_X$ is established

Me is any electroacoustic modifier that changes $L_D$ but not $L_R$ (*i.e.*, a shorter $D_2$)

N is the total acoustic power radiated by the system divided by the acoustic power radiated by the device or devices producing $L_D$ at $D_X$

$S\bar{a}$ is the total absorption in $ft^2$ (Sabins)

Ma is the architectural modifier $Ma = \left(\frac{1-\bar{a}}{1-a_c}\right)$

V is the internal volume of the enclosed space in $ft^3$

h is the height of the ceiling in ft

$RT_{60}$ is the '*apparent*' reverberation time in secs. for 60 dB of decay

$D_C$ is the critical distance (*i.e.*, distance at which the Hopkins-Stryker equation makes $L_D = L_R$) in ft

$$D_C = 0.141\sqrt{\frac{Q\,S\bar{a}\,(Me)(Ma)}{N}} = 0.03121^{*}\sqrt{\frac{QV\,(Ma)(Me)}{RT_{60}(N)}}$$

*Metric (S.I.) 0.057

0.734** a constant obtained by multiplying 0.221 by 3.322 (See *Sound System Engineering*, page 23)

**Metric (S.I.) 1.329

This constant allows calculation of the "LOG Multiplier"

$$\text{LOG Mult.} = (3.322)\,\Delta dB = 0.734\left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right)$$

## Notes on Above Data

I. Asymptotic limit for $D_X$ in Hopkins-Stryker equation is:

$$10 \text{ LOG} \left(\frac{4N}{S\bar{a}\ (Ma)}\right)$$

This is significant only when $D_X >>> D_C$ (*i.e.*, $D_X = 10\ D_C$)

II. English system sabins are $ft^2$. Metric (S.I.) sabins are $M^2$.

III. Derivation of the constant "10.5" in the Hopkins-Stryker equation when English system dimensions are employed.

$$\frac{0.282ft}{1} \cdot \frac{12\text{ in}}{1ft} \cdot \frac{2.54cm}{1\text{ in}} \cdot \frac{1M}{100\text{ cm}} = 0.08595M$$

$$20 \text{ LOG} \left(\frac{0.282M}{0.08595M}\right) = 10.3 \text{ dB}$$

Temperature and barometric pressure correction factor:

$$dB_{corr.} = -10 \text{ LOG} \left(\sqrt{\frac{^{\circ}F + 460}{527}} \left(\frac{30}{B}\right)\right)$$

Where: B is the barometric pressure in inches of HG

$^{\circ}F$ is the temperature in degrees Fahrenheit

67$^{\circ}$F and 30" of HG result in a correction factor of zero decibels

The additional 0.2 dB in the constant "10.5" allows for variation in standard temperature and pressure (STP)

IV. Originally as defined, $L_W$ allowed:

One acoustic watt from a source with a Q = 1.0 to produce a sound pressure level at 0.282 ft. (0.08595 M) of 130 dB (ref. $10^{-13}$ watt)

The current $L_W$ allows:

One acoustic watt from a source with a Q = 1.0 to produce a sound pressure level at 0.282 M (0.925 ft) of 120 dB (ref. $10^{-12}$ watt)

Thus, the ref. power was raised and the ref. dist. was increased.

$$10\ \mathrm{LOG}\left(\frac{10^{-12}W}{10^{-13}W}\right) = 10\ \mathrm{dB} \qquad 20\ \mathrm{LOG}\left(\frac{0.282M}{0.08595M}\right) = 10.3\ \mathrm{dB}$$

V. Further qualification of the Ma factor

$$Ma = \left(\frac{1-\bar{a}}{1-a_C}\right)\left(\frac{Q_{act}}{Q_{theor}}\right)$$

Where: $Q_{act}$ is the *measured* Q .

$Q_{theor}$ is the theoretical Q that the $C_L$ suggests (see *Sound System Engineering*, page 44)

$\bar{a}$ is the average absorption coefficient in the space

$a_C$ is the absorption coefficient of the area where the first reflection occurs. $a_C > \bar{a}$ must occur.

VI. Delta levels ($\Delta D_X$)

If $L_W$ is removed from the equations, they become $\Delta D_X$ equations yielding *relative* levels.

$$\Delta D_X = 20\ \mathrm{LOG}\left(\frac{0.282}{D_X < D_C}\right) + 10\ \mathrm{LOG}\ Q$$

Remembering that:

$$10\ \mathrm{LOG}\left(\frac{1.0}{4\Pi(0.282)^2} + \frac{4}{S\bar{a}}\right) \simeq 0$$

VII. Inverse functions

When $D_X < D_C$

$$D_X = \sqrt{\frac{Q}{4\Pi\left(10^{\left(\frac{\Delta D_X}{10}\right)}\right)}}$$

When $D_X \geqq D_C$

$$D_X = \left(10^{\left(\frac{\Delta D_X}{0.734\left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right)}\right)}\right) D_C$$

Basic Equation

$$D = \sqrt{\frac{Q}{4\Pi\left(10^{\left(\frac{\Delta D_X}{10}\right)} - \frac{4}{S\bar{a}}\right)}}$$

VIII. Other useful variations

$$S\bar{a} = \frac{(D_C)^2\ N}{0.019881\ Q\ (Ma)\ (Me)} \qquad Q = \frac{(D_C)^2\ N}{0.019881\ S\bar{a}\ (Ma)\ (Me)}$$

Also see *Sound System Engineering*, Appendix VIII, pages 261-270

When a $D_X$ value of 0.282 is used in the Hopkins-Stryker equation ($Q = 1$), it yields one square unit of area for the surface of a sphere of that radius in whatever dimension $D_X$ is expressed. If $D_X$ is in feet, then the area becomes 1 $ft^2$. If $D_X$ is in meters, then the area becomes 1 $M^2$ (0.282 ft = 0.08595m.) (0.282m = 0.925 ft.)

$$20 \text{ LOG } \left(\frac{0.282}{0.08595}\right) = 10.3 \text{ dB}$$

When the old standard reference for $L_W$ was $10^{-13}$ watt, one watt was an $L_W$ = 130 dB.

The new standard reference for $L_W$ is $10^{-12}$ watt (one picowatt), and one watt is an $L_W$ = 120 dB.

Since, in both cases, $L_W$ is a given *fact* when available, no power adjustment of the level $\Delta D_X$ is required. What is required is an adjustment in $\Delta D_X$ for the dimensional units ft or M. If meters are used, it is correct as written plus 0.2 dB for $L_W$ referenced to $10^{-12}$ watt. If ft are used, then 10.5 dB must be added to the equation as written because the distance is shorter. (10.3 + 0.2) = 10.5 dB.

When used to obtain $\Delta D_X$ numbers without $L_W$, no correction is required as the $\Delta D_X$ numbers are *relative* numbers. They become absolute levels only when used with an $L_W$.

## Peutz Modification of Hopkins-Stryker Equation

I. For $D_X$s under *apparent* $D_C$

$$\Delta D_X = 10 \text{ LOG } \left(\frac{Q}{4\pi (D_X)^2}\right)$$

II. For $D_X$s equal to or greater than *apparent* $D_C$

$$\Delta D_X = 10 \text{ LOG } \left(\frac{Q}{4\pi (D_C)^2}\right) + 0.734 \left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right) \left(\text{LOG}\left(\frac{D_C}{D_X \geq D_C}\right)\right)$$

III. Inverse calculations

When $D_X < D_C$

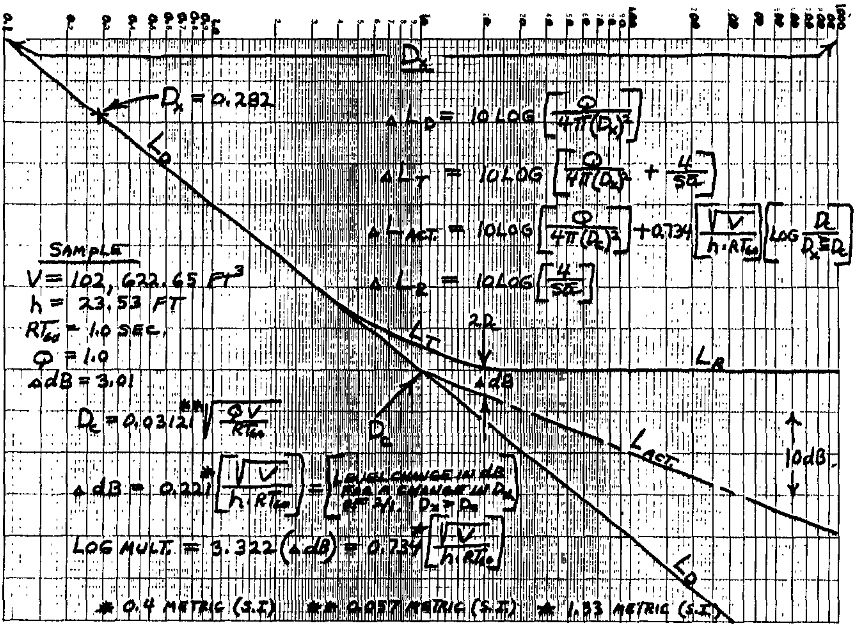
$$D_X = \sqrt{\frac{Q}{4\pi\left(10^{\left(\frac{D_X}{10}\right)}\right)}}$$

When $D_X \geq D_C$

$$D_X = \left(10^{\left(\frac{\Delta D_X}{0.734\left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right)}\right)}\right) D_C$$

Notes: Ref dist for 20 LOG$\left(\frac{ref}{D_X}\right)$ is 0.282

Apparent $D_C$ = 0.03121* $\sqrt{\frac{QV}{RT_{60}}}$ *0.057 metric




**Acoustic Level VS Distance**

## Describing Q More Accurately

The measurement of the directivity factor (Q) *is always at a point*. There can be a series of points within an area that have the same Q thus allowing the concept of an "average of Qs" within an area. It is a normal practice to measure Q on axes (the zero angle axis usually being the highest output as well). Let's call this measurement $Q_{axis}$.

The value Q is both frequency dependent, $Q_{axis}(f)$, and, for real life devices, angularly dependent. $Q_{axis}$ specifies the angle relative to the transducer. For angles other than the "on axis" position we could specify a $Q_{rel}$.

Wherein: $Q_{rel} = Q_{axis}\left(10^{\left(\frac{\pm C_{\angle}dB}{10}\right)}\right)$

Where: $\pm C_{\angle}dB$ indicates the level in dB of the particular angle *relative* to the level in dB on axis.

A complete descriptive may be specified by:

$$Q_{rel} = Q_{axis}\left(10^{\left(\frac{\pm C_{\angle}dB}{10}\right)}\right)(f)$$

Where: f is the frequency at which the measurement is made.

A further useful convention would be to agree that where no "f" is specified then the 1/3 octave band at 2000 Hz is indicated.

In the design of a sound system we use:

$Q_{min(ss)}$

Where: ss stands for single source and which *usually* is synonymous with $Q_{axis}$ but may, on occasion, actually be a $Q_{rel}$. The term *min* indicates that it is the minimum value that will allow the $\%AL_{cons}$ required at that *point*.

If more than one source is used, we encounter the term:

$NQ_{min}$

Wherein we increase the Q of the first device proportionately to the number (N) of additional devices (of equal acoustic power output).

We also employ the term $Q_{avail}$ whereby we can calculate the N required for a multiple source system.

$$N = \left(\frac{Q_{min}}{Q_{avail}}\right)$$

A further refinement is the direct calculation of a distance ($D_2$) at which the $Q_{avail}$ results in the same ratio of direct-to-reverberant sound as $NQ_{min}$ *would* have provided.

$$D_{2\ max} = \left(\frac{D_{2ss}}{N}\right)$$

At the current time we utilize the following Q descriptives:

$Q_{axis}$ $Q_{min}$ $Q_{rel}$ $Q_{avail}$

along with the descriptive modifiers: ss $\pm C_L dB$ N and f

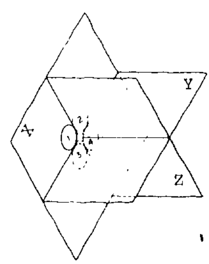
## A Subtlety Regarding "Q" by Placement

An often misinterpreted point with regard to establishing a directivity factor (Q) by placement of the source near a reflecting surface (mirror images) is that the source must be at, *not in*, the surface. (See left half of figure.)

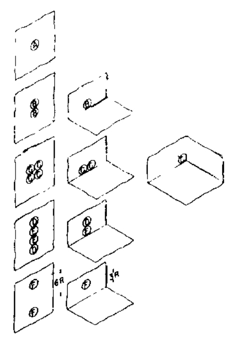
Loudspeakers mounted in the wall will, at lower frequencies, exhibit "mutual coupling" as shown in the right half of the figure.

When a single speaker is mounted *in* a wall, half the power goes into *another* space. When mounted near the wall, half the power is reflected back into the space.

From Henney's HANDBOOK OF ENGINEERING




Primary images 2, 3, and 4 of piston 1 introduced by planes Y and Z.



Effect of adding pistons and reflecting planes on radiation impedance. All pistons marked with the same letter see the same radiation impedance.

## $\Delta D_X$ and the Use of Q

Users of the Hopkins-Stryker equation often question whether different Qs should be employed for $D_S$, $D_1$, $D_2$ and $D_0$ when obtaining $\Delta D_X$. Normally, the answer is "use the loudspeaker Q for all distances." Differing Qs (for example, the talker with a Q of 2.5 and the loudspeaker with a Q = 50) will establish quite different $D_C$s.

In the normalized gain equations, $D_0$ drops out and is replaced by EAD. This means that both the $\Delta D_S$ and $\Delta$EAD are *normally* in the direct sound field where inverse-square-law level change is the rule and the Q chosen can be arbitrary *so long as it is the same for both distances*. In any case, the remaining two distances, $D_1$ and $D_2$, are dependent upon the loudspeaker's Q.

## How to Use Differing Values

If it is desired to use separate Q values for the talker and loudspeaker (and perhaps to assign a Q to the microphone as well), you are then freed to obtain an absolute level change rather than a relative one. This is accomplished by using a reference point and a $D_X$ point for each value and taking the $\Delta D_X$s of both points, followed by using the difference between them as the $\Delta D_X$ in the gain equations.

What you may not legitimately do is use differing Qs in the Hopkins-Stryker equation when obtaining *relative* $\Delta D_X$s. The reference point chosen should usually be less than 0.5 feet.

## Using the Hopkins-Stryker Equation

The sound power level ($L_W$) is referenced to $10^{-12}$ watt. In using the Hopkins-Stryker equation to obtain an expected sound pressure level (Lp) at some distance ($D_X$), it is important not to use the $L_W$ of the array but only the $L_W$ of that part of the array supplying $L_D$ and $D_X$. To do otherwise is to miscalculate the $L_D$ which is dependent *only* upon the $L_W$ of the devices also producing the $L_D$ at the point of observation (measurement). The N factor inserted into the equation in opposition to the total absorption ($S\bar{a}$) correctly adjusts for the ratio of total $L_W$ to the $L_W$ producing $L_D$ because this portion of the Hopkins-Stryker equation affects only the reverberant sound field level ($L_R$).

Always bear in mind that $L_D$ is affected by that part of $L_W$ producing $L_D$ at the point of measurement ($D_X$) distance from the array, the Q of the device producing $L_D$ at $D_X$ (not the "Q" of the array), and the distance from the array ($D_X$). Thus, we avoid the difficulties of overestimating the level of $L_D$ at $D_X$.

$L_R$ is affected by the *total* $L_W$ of the array (which may be properly accounted for by the ratio N which scales the level appropriately to the $L_W$ of the single device producing $L_D$) and the total absorption present. Here it is important to note the sometimes significant role of the architectural acoustic modifier ($M_a$). A substantial $M_a$ can lower $L_R$ (but $RT_{60}$ or the decay rate remains the same). The $M_a$ factor is invariably lower than would be expected because of the difference between the Q of real life devices and the coverage angles employed. It is important to note that this effect cannot operate unless $a_c > \bar{a}$.

## Summary

These instructive equations reveal the interaction of each of the primary parameters controlling the various sound fields. We have discussed additional parameters that have direct bearing on the modified behavior of the primary parameters. In its modified form, the Hopkins-Stryker equation has kept pace with measurements in the sense that our prediction accuracy has kept pace with our measurement capability.